12.812: Problem set 3

Need help?: Office hours Wednesday 11am-noon; Also email or drop by as needed.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. **Mountain torque** We showed in class that the zonal-mean torque on the atmosphere due to mountains results from the difference in surface pressure from the east to the west side, and that it may be written as

$$\frac{1}{2\pi}\int_{z_{\min}}^{\infty}\sum_i\left(p_s^{Ei}-p_s^{Wi}\right)dz,$$

where $z_{\min}$ is the minimum in surface geopotential height at the latitude in question, p_s^{Ei} and p_s^{Wi} are the time-mean surface pressures at the intersection of mountain i with level z on the eastern and western sides, respectively, and $\sum_i$ is a sum over all the mountains that reach z at that latitude. While this formula makes intuitive sense, it is difficult to evaluate in practice. Show that it may be rewritten in terms of surface geopotential height gradient and surface pressure as

$$-\left[p_s\frac{\partial z_s}{\partial \lambda}\right],$$

where $[\cdot]$ denotes the zonal mean, p_s is time-mean surface pressure, z_s is surface geopotential height, and λ is longitude. (Hint: rewrite $p_s^{Ei}-p_s^{Wi}$ as $\int_{\lambda_W}^{\lambda_E}\frac{\partial p_s}{\partial \lambda}d\lambda$ and use a Heaviside function in $z_s - z$ to extend the λ integral to all longitudes.)

2. **Rossby wave chromotography** Consider a barotropic zonal jet in midlatitudes on a beta plane of the form

$$[u(y)] = u_0\exp\left(-\frac{y^2}{2L_y^2}\right)$$

where u_0 and L_y are constants and [] indicates the zonal mean. Suppose there is a source of wave activity at $y = 0$ that generates waves propagating equally in the north and south directions. For simplicity, the cospectrum of the wave velocities u* and v* is assumed to be constant as a function of phase speed and zero for negative phase speeds. The waves are assumed to be of small amplitude.

(a) Calculate the tendency of [u(y)] at $y \neq 0$ due to the momentum flux of the waves. Include a schematic of the momentum fluxes and wave propagation in your answer.

(b) Now suppose there is a turning latitude at $y = L_y$. For simplicity the turning latitude is assumed to be the same for all phase speeds. Recalculate the tendency of [u(y)] taking the turning latitude into account. Include a schematic of the fluxes and wave propagation in your answer.

3. **Response of relative humidity to global warming** Consider an idealized climate-change scenario in which the tropospheric temperature change is spatially uniform and the circulation does not change. Make an estimate of the change in relative humidity (for a 1K warming) owing to the fact that saturation vapor pressure is not exactly an exponential function of temperature. Is this change in relative humidity large compared to the fractional change in saturation vapor pressure?